

Resolving the wave-vector and the refractive index from the coefficient of reflectance

V. U. Nazarov* and Y.-C. Chang

*Research Center for Applied Sciences, Academia Sinica, 128 Section 2 Academia Road,
Nankang, Taipei 115, Taiwan*

**Corresponding author: nazarov@gate.sinica.edu.tw*

We resolve the existing controversy concerning the selection of the sign of the normal-to-the-interface component of the wave-vector k_z of an electromagnetic wave in an active (gain) medium. Our method exploits the fact that no ambiguity exists in the case of a *film* of the active medium since its coefficient of reflectance is invariant under the inversion of the sign of k_z . Then we show that the limit of the infinite film thickness determines a unique and physically consistent choice of the wave-vector and the refractive index. Practically important implications of the theory are identified and discussed.

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Recently, there has been much of debate regarding the correct selection of the sign of the normal-to-the-interface component of the wave-vector k_z of an electromagnetic wave propagating in an active medium or, equivalently for the normal incidence, of the refractive index of the active medium [1–6]. Specifically, if one considers a system of a glass prism, a metal film, and an active dielectric (the Kretschmann-Raether configuration) as schematized in Fig. 1 (a), then the coefficient of reflectance is given by [7] (for the sake of simplicity, we consider nonmagnetic media with $\mu = 1$)

$$R = \left| \frac{r_{01} + r_{12}e^{2ik_{1z}d_1}}{1 + r_{01}r_{12}e^{2ik_{1z}d_1}} \right|^2, \quad (1)$$

where the coefficients

$$r_{ij} = (k_{iz}\epsilon_j - k_{jz}\epsilon_i)/(k_{iz}\epsilon_j + k_{jz}\epsilon_i) \quad (2)$$

describe the parallel field components,

$$k_{iz} = \pm \frac{2\pi}{\lambda} \sqrt{\epsilon_i - \epsilon_0 \sin^2 \theta}, \quad (3)$$

ϵ_0 , ϵ_1 , and ϵ_2 are complex dielectric constants of the prism, the metal, and the active dielectric, respectively, λ is the light wave-length in vacuum, and θ is the angle of incidence. For an active medium ($\epsilon_2'' < 0$) the choice of the sign of the square root in Eq. (3) is far from evident [1–6,8] while this choice strongly affects the coefficient of reflectance of Eq. (1).

In this Letter, we show that the problem of the sign of k_{2z} can be unambiguously and naturally resolved by first considering a film of the active dielectric then letting the thickness of the film tend to infinity. Let us consider a system in Fig. 1 (b) which differs from that in Fig. 1 (a) by the finite thickness d_2 of the active dielectric and the presence of a semi-infinite passive dielectric with the dielectric constant ϵ_3 . For this system the coefficient of reflectance can be written as [9]

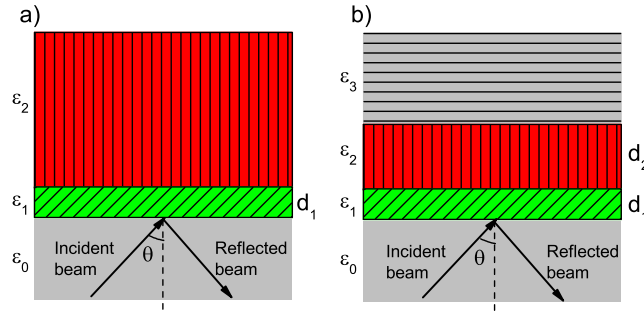


Fig. 1. (Color online) Systems under study: (a) Semi-infinite dielectric (ϵ_0), metal film (ϵ_1 , d_1) and a semi-infinite active medium (ϵ_2). (b) As in (a) but the active medium constitutes a film of the thickness d_2 , and there is a semi-infinite dielectric on the top with the dielectric function ϵ_3 .

$$R = \left| \frac{e^{2ik_{1z}d_1} (r_{12} + e^{2ik_{2z}d_2} r_{23}) + r_{01} (1 + e^{2ik_{2z}d_2} r_{12} r_{23})}{1 + e^{2ik_{2z}d_2} r_{12} r_{23} + e^{2ik_{1z}d_1} r_{01} (r_{12} + e^{2ik_{2z}d_2} r_{23})} \right|^2. \quad (4)$$

This is evident [and can also be directly verified with use of Eq. (2)] that Eq. (4) is invariant under the transformation $k_{2z} \rightarrow -k_{2z}$. Therefore, there is no ambiguity in the selection of the sign of k_{2z} in the case of a film of the active dielectric. Now, putting d_2 to infinity in Eq. (4), we immediately retrieve Eq. (1) if and only if k_{2z} has a positive imaginary part *regardless of whether medium 2 is active or passive*. Accordingly, the branch cut in the complex plane of k_{2z}^2 must be taken along the *positive part of the real axis* ($0 \leq \phi < 2\pi$, where ϕ is the argument of k_{2z}^2), in agreement with [2] and in disagreement with [1] and [3,10], where the branch cuts were taken along the negative imaginary ($-\pi/2 \leq \phi < 3/2\pi$) and negative

real ($-\pi \leq \phi < \pi$) axes, respectively. Therefore, for an active medium the refractive index $n = \sqrt{\epsilon}$ has negative real part. This concludes our proof as which of the two signs of k_{2z} should be chosen in Eq. (3) to be substituted into Eq. (1).

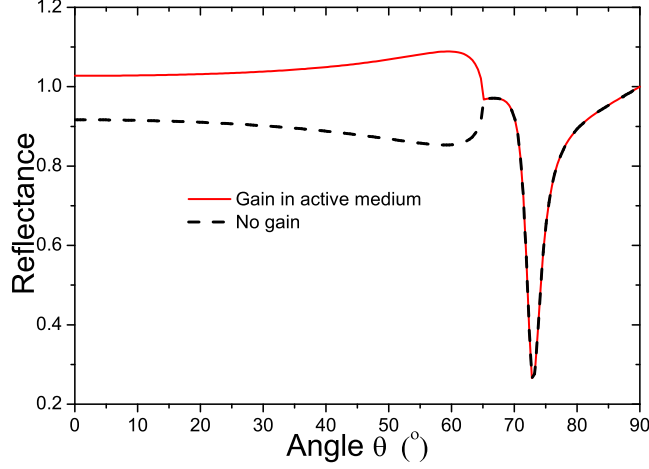


Fig. 2. (Color online) Coefficient of reflectance of the system of a glass prism, silver film, and the dye of cresyl violet with ($\epsilon_2 = 1.85 - 9 \times 10^{-6}i$, red solid line) and without ($\epsilon_2 = 1.85 + 0 \times i$, black dashed line) gain. Other parameters are those from [11] (see text).

In order to provide an illustrative example which helps to elucidate a number of practically instructive points, in Figs. 2 - 4 we present numerical results for the coefficient of reflectance of the system of a glass prism, a silver film, and the dye of cresyl violet. Parameters as taken from [11] are: $\epsilon_0 = 2.25$, $\epsilon_1 = -18 + 0.7i$, $\epsilon_2 = 1.85 - 9 \times 10^{-6}i$, $d_1 = 39$ nm, and $\lambda = 633$ nm.

In Fig. 2 the coefficient of reflectance of Eq. (1) of the system with the semi-infinite active dielectric (red solid line) is plotted and compared with that of a passive dielectric (black dashed line). The sharp dip in the spectrum with the minimum at 73.1° is due to the surface plasmon polariton associated with silver film. At lower angles of incidence, the reflectance is mainly greater than one for the system with the active dielectric, obviously due to the gain in the latter. It must be noted that the value of $\epsilon_2'' = -9 \times 10^{-6}$ is small enough that the corresponding spectrum is indistinguishable from that of the system with $\epsilon_2'' = 0_-$, where 0_- stands for the zero limit from the negative side. An important point is that there is no contradiction in reflectance of the systems with $\epsilon_2'' = 0_+$ and $\epsilon_2'' = 0_-$ to differ finitely (and considerably) as is seen in Fig. 2, since $\epsilon_2'' = 0_-$ combined with *infinite* thickness of the active medium results in the finite overall gain (mathematically, the limit of infinite thickness should be taken first at finite negative ϵ_2'' , then the latter should be put to zero).

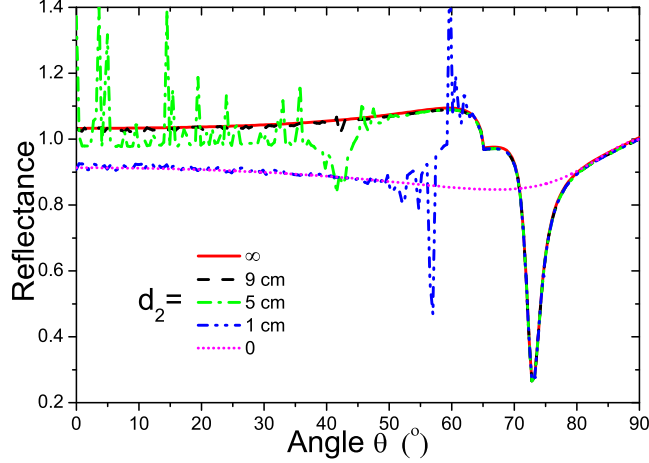


Fig. 3. (Color online) Convergence of the coefficient of reflectance of the system with a finite film of the active dielectric to that with the semi-infinite one. Parameters are those from [11] (see text).

In order to understand what films of the active dielectric are thick enough to be considered as semi-infinite, in Fig. 3 we plot the coefficient of reflectance of Eq. (4) for a number of film thicknesses. The striking result of the saturation of the spectrum of a film to that of the semi-infinite medium occurring at the film thickness as huge as about 9 cm is simply due to the tiny $\epsilon_2'' = -9 \times 10^{-6}$, and this should serve as a warning when weakly active dielectric films are modeled with semi-infinite media.

In Fig. 4 we plot the reflectance of Eq. (1) obtained with three different branch cuts in the complex plane of k_{2z}^2 . These are compared with the coefficient of reflectance of the system with a film of active dielectric ($d_2 = 9$ cm), for which the selection of the branch cut does not make a difference, and which, therefore, at large film thickness can serve as a criterion of the validity of the branch cut chosen in the semi-infinite case. In agreement with our formal proof, the spectrum calculated with the branch cut along the positive real axis ($\text{Im } k_z > 0$) (red dashed line) practically coincides at all angles with that of the film (black solid line). At the same time, the cut along the negative imaginary axis [1] (green dashed line) results in a loss instead of the gain below the critical angle, while the cut along the negative real axis [3, 10] (blue dashed line) leads to the incorrect reflectance in the range of the surface plasmon polariton.

We point out a two-fold significance of deriving the wave-vector in a half-space from that for a finite film: Firstly, this method provides a mathematically rigorous and simple procedure to resolve the sign of the wave-vector. Secondly, in a more general perspective, in experiment there evidently exists no semi-infinite medium but only (maybe thick) films. A half-space is

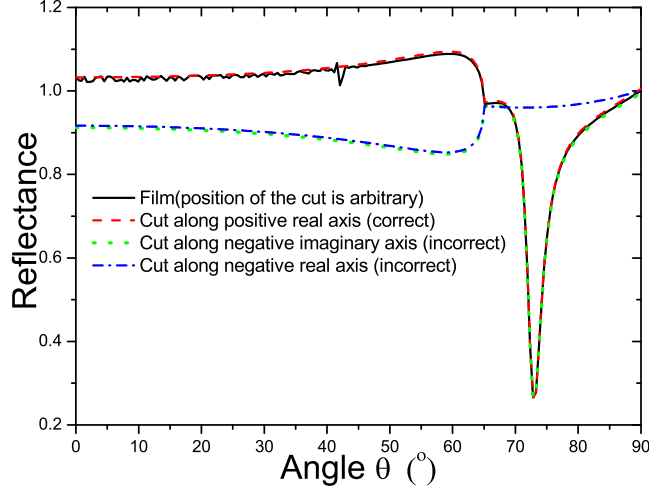


Fig. 4. (Color online) Implications of the selection of the three different branch cuts in k_z^2 complex plane. Red dashed line corresponds to the branch cut along positive real axis ($0 \leq \phi < 2\pi$). Green dotted line corresponds to the branch cut along negative imaginary axis ($-\pi/2 \leq \phi < (3/2)\pi$) [1]. Blue dashed-dotted line corresponds to the branch cut along negative real axis ($-\pi \leq \phi < \pi$) [3, 10]. Black solid line corresponds to a film of the active dielectric ($d_2 = 9$ cm). Other parameters are those from [11] (see text).

a convenient abstraction with properties being the limit of the corresponding properties of a film thick enough so that a further increase of the film's thickness does not change results of an experiment. Therefore, our results for semi-infinite active media are automatically applicable to the interpretation of experimental data with sufficiently thick films. This is not the case with other choices of the branch cut in the complex plane of k_{2z} [1, 3, 10] as has been already discussed in conjunction with Fig. 4.

Ref. [12] performs a numerical analysis of a wave-packet propagation in a system considered in [2]. Correcting errors of [3] by the same authors, [12] reconfirms the conclusion of [2] that the refractive index of the active medium considered in the latter reference has a positive imaginary part. This conclusion is in agreement with results of the present work. In particular, [12] does not find the pulse refracted into the active medium to grow unboundedly which perfectly confirms the positive imaginary part of the refractive index.

In conclusion, by considering a semi-infinite active (gain) dielectric as a limiting case of its finite-thickness film, we have resolved the existing controversy in determining the normal-to-the-interface component of the wave-vector k_z and, equivalently, of the sign of the refractive index of the active dielectric. Specifically, $\text{Im } k_z$ must be nonnegative for both active and

passive media, and, accordingly, the refractive index of an active medium has a negative real part. We have shown that this is physical that the coefficient of reflectance can be discontinuous with respect to a continuous change of the dielectric constant of a semi-infinite medium from that of a passive to an active one. We have applied the theory to the system of a glass prism, silver film, and the weakly active dye of cresyl violet in Kretschmann-Raether configuration, having numerically illustrated the above general concepts.

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